Simple Harmonic Motion

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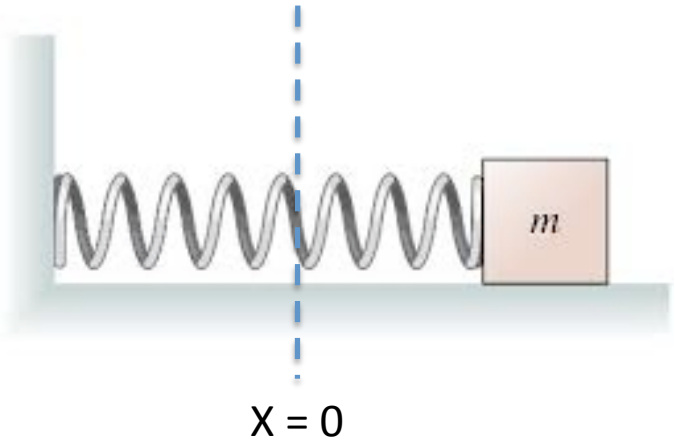
- When a Hooke's law force acts on an object, that object will oscillate back and forth in SHM.

$$F = -kx$$

- Consider a mass attached to a spring



What is the position of the mass as a function of time?

$$F = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-k}{m}x$$ * let's call k/m ω^2

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Can you think of a function x so that when you take its 2nd derivative you get that function again but with ω^2 out in front?

General Solution

$$x = A\cos(\omega t)$$

$$v = x' = -\omega A\sin(\omega t)$$

$$a = v' = -\omega^2 A\cos(\omega t)$$

Helpful Shortcuts:

$$x_{\max} = A$$

$$v_{\max} = \omega A$$

$$a_{\max} = \omega^2 A$$

What Does ω Measure?

$$\omega = \sqrt{\frac{k}{m}}$$

- ω measures angular frequency
- Units = [**rad/s**]
- If k is large (stiff spring) and mass is small, it will oscillate back and forth very frequently.

- We can also express the frequency in cycles per second (Hz).
- 1 cycle = 2π rad

$$\omega = 2\pi f$$

- Period and frequency have an inverse relationship.

$$T = \frac{1}{f}$$

Ex1: If ω = 10π rad/s, what is f? T?

Ex2: A 2 kg mass oscillates with a displacement given by $x = 3\cos(4\pi t + \pi)$

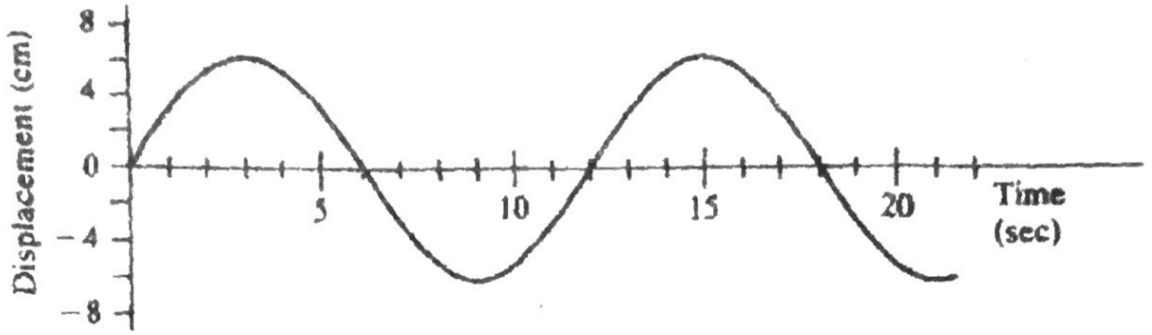
What is T? k? v_{max}? $a_{max?}$

Phase constant
(shifts starting point of graph)

Introduction to Simple Harmonic Motion

Each of the graphs below is a displacement vs. time graph for an object that undergoes SHM.

1)



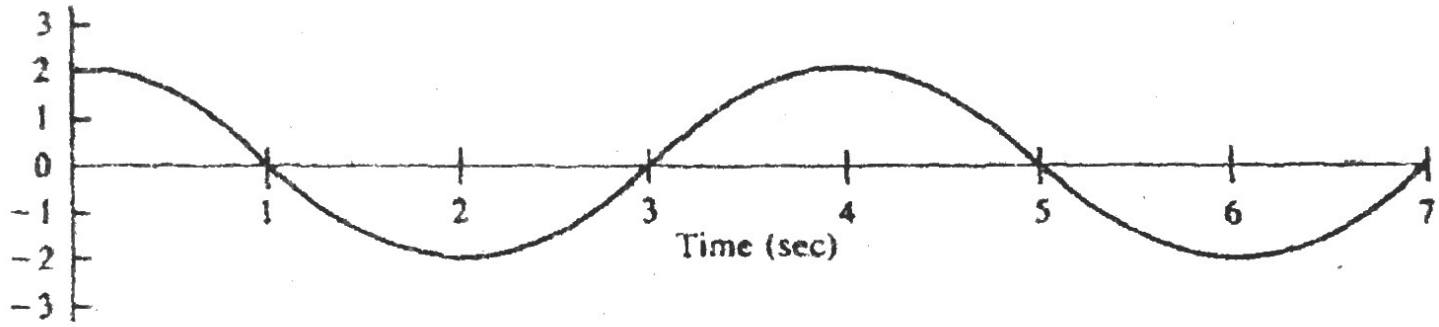
What can you find by inspecting this graph?

Write an equation that gives the displacement as a function of time.

Write an equation that gives the velocity as a function of time.

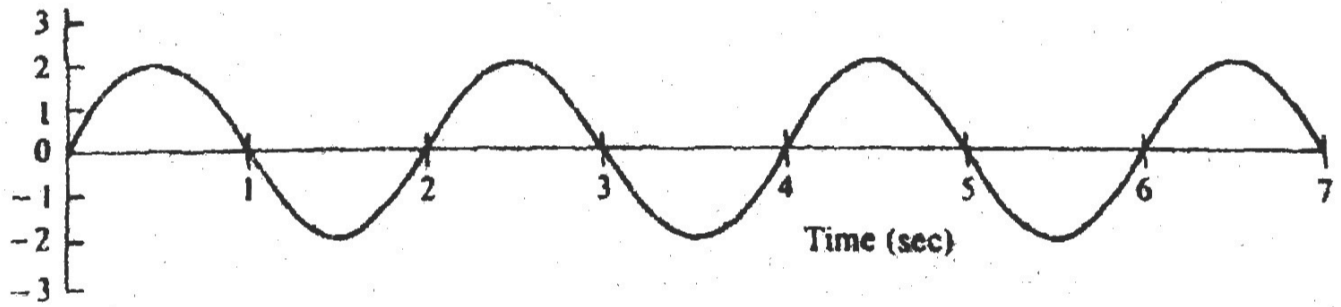
Write an equation that gives the acceleration as a function of time.

2)



What is the displacement, velocity, and acceleration of this object at t = 4 s?

3)



At what time does the maximum velocity first occur? What is the maximum velocity?

At what time does the maximum acceleration first occur? What is the maximum acceleration?

How far does this oscillator travel in one cycle?