

Moment of Inertia

& Parallel Axis Theorem

Rotational Inertia (Moment of Inertia)

- Measure of resistance a rigid object has to a change in its angular velocity.
- Describes how the mass of an object is distributed about its axis of rotation.



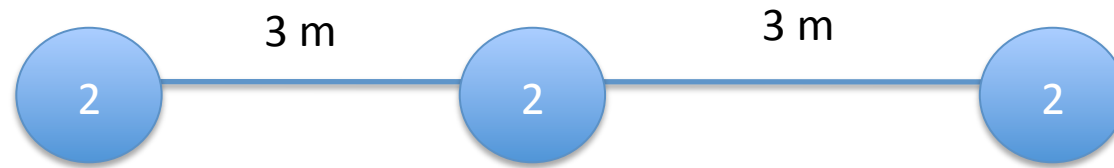
Calculating Rotational Inertia

- For **point masses**, sum up the individual masses multiplied by their distance from the axis of rotation squared.

$$I = \sum m_i r_i^2$$

unit = kgm^2

Ex: Each mass shown below is 2 kg. The rod that connects them is 6 m long and has negligible mass. Calculate the rotational inertia of the system if it is rotated about:



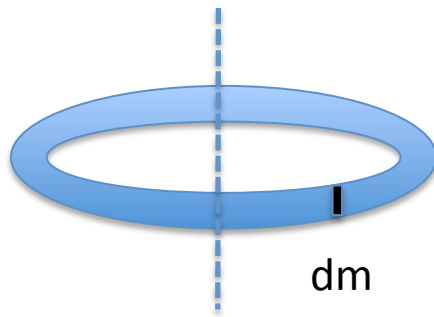
one end:

the middle:

- For **rigid objects**, find the product mr^2 for each particle and then sum the products. Take the limit of the sum as $\Delta m \rightarrow 0$.

$$I = \int r^2 dm$$

Ex: Find the rotational inertia of a thin hoop of mass M and radius R rotating about an axis through its center.

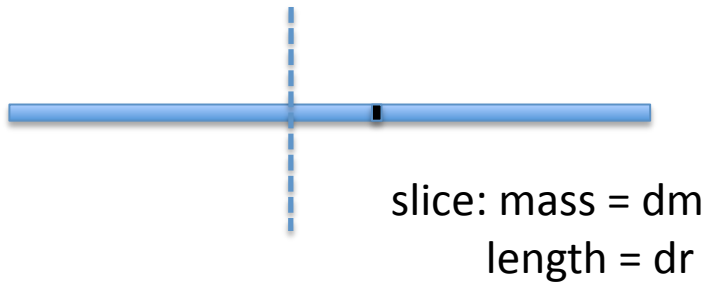


$$I = \int_0^M r^2 dm$$

$$= R^2 \int_0^M dm$$

$$I = MR^2$$

Ex: Find the rotational inertia of a thin uniform rod of mass M and length L rotating about an axis through its center and perpendicular to it.



$$I = \int_0^M r^2 dm$$

Rod has uniform density so: $\frac{M}{L} = \frac{dm}{dr}$

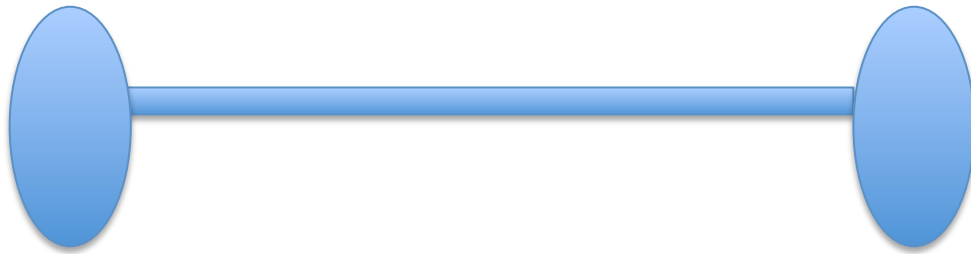
$$dm = \frac{M}{L} dr$$

$$I = 2 \int_0^{\frac{L}{2}} \frac{M}{L} r^2 dr$$

$$I = 2 \frac{M}{L} \frac{r^3}{3} \bigg|_0^{\frac{L}{2}}$$

$$I = \frac{1}{12} ML^2$$

Ex: Find the rotational inertia of the barbell shown below that rotates about an axis through its center. The bar has a mass of 10 kg and a length of 1.2 m. Each weight on the end has a mass of 20 kg.



Commonly Used Moments of Inertia

(all are rotating about their center of mass)

Hoop	Cylinder/Disk
$I = MR^2$	$I = \frac{1}{2} MR^2$
Thin Rod	Sphere
$I = \frac{1}{12} MR^2$	$I = \frac{2}{5} MR^2$

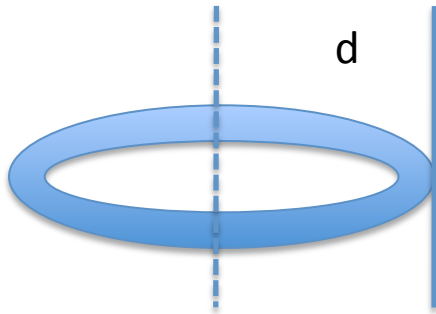
What do we do if the axis of rotation doesn't pass through the center of the object?

Parallel Axis Theorem

$$I = I_{COM} + Md^2$$

Where d is the distance between the given axis and the axis through the center of mass.

Ex: Find the rotational inertia of a hoop of mass M and radius R rotating about an axis tangent to its side.



$$I = MR^2 + Md^2$$

$$I = MR^2 + MR^2$$

$$I = 2MR^2$$

Where $d=R$

Ex: Find the moment of inertia of a thin rod of mass M and length L rotating about an axis through one of its ends. Solve using 2 methods.



1. Parallel Axis Theorem

$$I = I_{COM} + Md^2$$

2. Integrate

$$I = \int_0^L \frac{M}{L} r^2 dr$$

Kinetic Energy

- If linear (translational) kinetic energy is $\frac{1}{2}mv^2$ then rotational kinetic energy is $\frac{1}{2}I\omega^2$
- An example of an object that has just *rotational kinetic energy* is a spinning CD.
- An example of an object that has just *translational kinetic energy* is a box sliding across the ground.
- An an example of an object that has *both rotational and translational kinetic energy* is a wheel rolling down a hill.