Torque Notes

To make an object start rotating about an axis clearly requires a force. But the direction of this force, and where it is applied, are also important.

Say you want to push open a door. It will be difficult for you to do this if you apply your force near the hinge. The reason that doorknobs are always as far from the hinge side of the door as possible is because your force is more effective when it is applied farther from the axis of rotation.

This is the same principle that allowed the 100-gram mass to balance the 200-gram mass in the lab. By placing the 100-g mass twice as far from the axis of rotation as the 200-g mass, it was able to provide the same amount of torque as the heavier object.

Torque is the product of force x lever arm, where lever arm is the distance between the force and the axis of rotation. The unit of force is the newton-meter (Nm). (Even though 1 J = 1 Nm, we never say that torque comes in joules.) The direction of torque is clockwise (CW) or counterclockwise (CCW).

$$\boldsymbol{\tau = F \times d}$$

Example

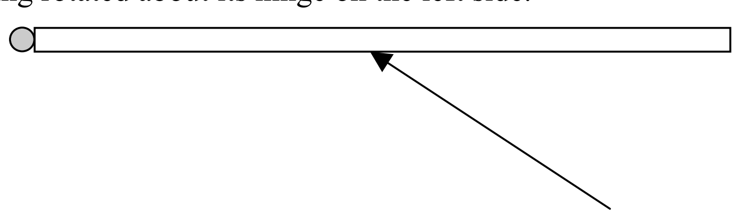
A 15 kg mass is placed on a seesaw 30 cm to the left of the fulcrum. Where can a 5-kg mass be placed to balance the seesaw?

In the torque lab, many groups correctly figured out that $m_1d_1 = m_2d_2$. Technically this should be $F_1d_1 = F_2d_2$, but since we can divide through by g, the first equation works for balancing a seesaw with weights.

$15g(.30) = 5g(x)$
$x = .90$ m (to the right of the fulcrum)

The 15-kg mass exerts a CCW torque on the seesaw. In order to keep the seesaw in equilibrium, we need the net torque to be zero. Therefore the second torque should have the same magnitude as the first but be CW. To rotate the seesaw CW the second mass should be placed to the right of the fulcrum.

The direction of force also matters. The diagram below shows a bird's eye view of door being rotated about its hinge on the left side.



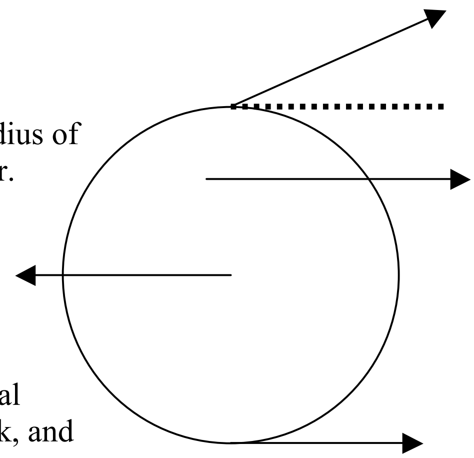
This force can be broken down into two components – the amount perpendicular to the lever arm, and the amount parallel to the lever arm. Only the perpendicular component exerts torque. Torque is a cross product. (Unlike work which was a dot product.) To find the magnitude of torque, multiply force by lever arm and then **multiply by sine of the angle between force and lever arm**. This will give you the amount of force that is in the perpendicular direction. In the case of balancing the seesaw in the lab, the direction of force was down (perpendicular to the lever arm) so it was unnecessary to multiply by sine of 90º.

More Examples

1. A person exerts a force of 28 N on the end of a door that is 84 cm wide. What is the magnitude of the torque if the force is exerted (a) perpendicular to the door, and (b) at a 60º angle to the face of the door?

   (a) 28(0.84) sin(90) = **23.5 Nm**
   (b) 28(0.84) sin(60) = **20.4 Nm**

2. The disk shown to the right has a mass of 20 kg and a radius of 3.0 m and is free to rotate about an axis through its center. Four forces act on the disk as shown:
   7 N at r = 0 m,
   10 N at r = 2.0 m,
   6 N at r = 3.0 m
   8 N at r = 3.0 m at 30° to a line tangent to the disk.
   Determine (a) the net torque on the disk, (b) the rotational inertia of the disk, (c) the angular acceleration of the disk, and (d) the tangential acceleration of a point on the rim of the disk?



   (a) $\tau_1 = 0$, $\tau_2 = 20$ CW, $\tau_3 = 18$ CCW, $\tau_4 = 12$ CW, net torque = 14 Nm CW
   (b) $I = ½ mr^2 = ½ (20)(3)^2 =$ **90 kgm²**
   (c) $\Sigma\tau = I\alpha \rightarrow 14 = 90\alpha$, $\alpha =$ **0.156 rad/s²**
   (d) $a_t = \alpha r = 0.156(3) =$ **0.467 m/s²**

3. A young mom pushes tangentially on a small merry-go-round and is able to accelerate it from rest to a spinning rate of 30 rpm in 10.0 s. Assume the merry-go-round is a disk of radius 2.5 m and has a mass of 800 kg, and two children (each with a mass of 25 kg) sit opposite each other on the edge. Find (a) the moment of inertia of the merry-go-round with children onboard, (b) the angular acceleration of the merry-go-round, (c) the minimum force required to achieve that torque. Ignore friction.

   (a) $I = I_{disk} + I_{kid} + I_{kid} = ½(800)(2.5)^2 + (25)(2.5)^2 + (25)(2.5)^2 =$ **2813 kgm²**
   (b) $\omega = 30$ rpm x 2π rad/ 60 s = 3.14 rad/s,
   $\omega = \omega_o + \alpha t \rightarrow 3.14 = 0 + \alpha(10)$, $\alpha =$ **.314 rad/s²**
   (c) $\Sigma\tau = I\alpha \rightarrow F(2.5) \sin(90) = 2813(.314)$, F = **353 N**

4. An 80-gram meter stick is bolted to a wall on one end and supported by a thin string on the other end so that it lies parallel to the ground. The string is cut and the meter stick rotates about the bolted end. At the moment the string is cut, what is the angular acceleration of the meter stick?

   $\Sigma\tau = I\alpha$
   $.080(9.8)(0.50) = 1/3\ (.080)(1^2)\ \alpha$
   $\alpha =$ **14.7 rad/s²**