Kinematics

With Calculus
Non-Constant Acceleration

• When acceleration is not constant, it is difficult to analyze motion graphically, and the formulas for "Kinematics with constant acceleration" are NOT valid.

• In these cases, we can use calculus to generate position, velocity, or acceleration functions.

• Reminder: The Power Rule for taking derivatives

\[
\frac{d}{dt}(kx^n) = nkx^{n-1}
\]

**Ex:** Find $dx/dt$

\[
x = 5t^3 + 67t^2 + 3t + 89
\]
Integrals

- Integrals are the reverse of derivatives.
- Purpose: Given the slope of a curve, find the formula for the curve itself.
- Geometrically, integrals find the area under curves.

Ex: Given \( \frac{dy}{dt} = 10t \) find the equation of \( y \) as a function of time.

Given: At \( t = 4 \), \( y = 100 \). Find the constant of integration.
**Key:** Time is always on the horizontal axis. The graphs will be plotted in groups of three. X on top, then V, finally A.

- Take the slope (derivatives)
- Find the area under the curve (integrals)
Kinematics Formulas

\[ v = \frac{dx}{dt} \quad \quad x = \int (v) \, dt \]

\[ a = \frac{dv}{dt} \quad \quad v = \int (a) \, dt \]

**Ex:** Given \( x = 3t^4 \) find formulas for velocity and acceleration.

**Ex:** Given \( a = 60t^2 \) at \( t=2, \ v = 250 \). At \( t=3, \ x=400 \). Find formulas for \( v \) and \( x \).

More examples on “Calculus Based Motion Problems” handout