Kinematics

With Calculus

Non-Constant Acceleration

- When acceleration is not constant, it is difficult to analyze motion graphically, and the formulas for "Kinematics with constant acceleration" are NOT valid.
- In these cases, we can use calculus to generate position, velocity, or acceleration functions.

- Reminder: The Power Rule for taking derivatives

$$\frac{d}{dt}\left(kx^{n}\right) = nkx^{n-1}$$

Ex: Find dx/dt

$$x = 5t^{3} + 67t^{2} + 3t + 89$$

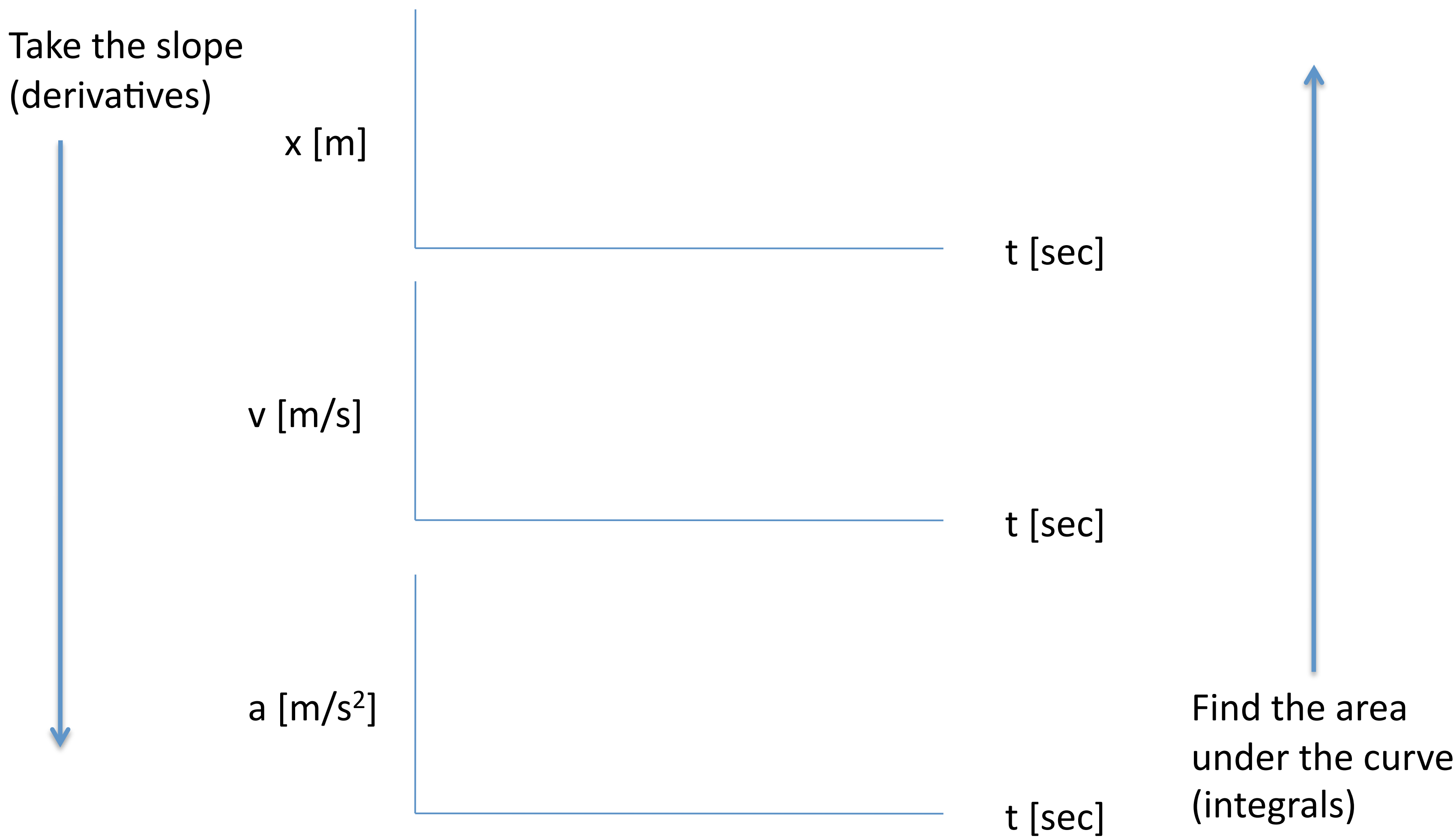
Integrals

- Integrals are the reverse of derivatives.
- Purpose: Given the slope of a curve, find the formula for the curve itself.
- Geometrically, integrals find the area under curves.

Ex: Given $\frac{dy}{dt} = 10t$ find the equation of y as a function of time.

Given: At t = 4, y = 100. Find the constant of integration.

Key: Time is always on the horizontal axis. The graphs will be plotted in groups of three. X on top, then V, finally A.



Kinematics Formulas

$$v = \frac{dx}{dt} \qquad x = \int (v)\,dt$$

$$a = \frac{dv}{dt} \qquad v = \int (a)\,dt$$

Ex: Given $x = 3t^4$ find formulas for velocity and acceleration.

Ex: Given $a = 60t^2$ at t=2, v = 250. At t=3, x=400. Find formulas for v and x.

More examples on "Calculus Based Motion Problems" handout