Gravitational PE Function

The Real Gravitational PE Function

- Recall, the relationship between a conservative force and its potential energy function is: $F_r = -\frac{dU}{dr}$ $\quad U_r = -\int F_r dr$
- The force of gravity is $\frac{-Gm_1m_2}{r^2}$ (where – sign indicates the force is attractive)
- Therefore, the PE function is

$$U_r = -\int \frac{-Gm_1m_2}{r^2} dr = Gm_1m_2 \int \frac{1}{r^2} dr = -\frac{Gm_1m_2}{r}$$

- So, we have negative potential energy.
- Where will we have zero potential energy? At ∞

$$\boxed{U = -\frac{GmM}{r}}$$

- Let's say I move a book from the floor to height h. What is the change in the potential energy?

$$\Delta U = U_f - U_i$$

$$= -\frac{Gm_1m_2}{R_E + h} + \frac{Gm_1m_2}{R_E}$$

$$= -\frac{Gm_1m_2R_E}{(R_E + h)R_E} + \frac{Gm_1m_2(R_E + h)}{R_E(R_E + h)}$$

$$= \frac{Gm_1m_2h}{(R_E + h)R_E}$$

If $h \ll R_E$

$$\Delta U = \frac{Gm_1m_2h}{R_E^{\ 2}}$$

$$\Delta U = mgh$$

When an object is lifted, it gains potential energy because it has less negative energy. This is a positive change.

- This is why the potential energy <u>near the surface of the earth</u> = mgh.

Escape Speed

- How fast must you fire a projectile so that it can completely escape Earth's gravitational pull?
- If it *just* escapes, it will get to ∞ with zero speed.
- It will have zero energy there, so by conservation of energy:

$$K + U = 0$$

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0$$

$$v^2 = \frac{2GM}{r}$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

Orbital Energy

- A body of mass m has a circular orbit of radius r. What is its total energy?

$$E = K + U$$

$$= \frac{1}{2}mv^2 - \frac{GmM}{r}$$

$$= \frac{1}{2}m\left(\frac{GM}{r}\right) - \frac{GmM}{r}$$

$$\boxed{E = -\frac{GmM}{2r}}$$

$$F_c = m\frac{v^2}{r}$$

$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

An object with (-) energy is bound.